*Indian J. Phys.* **51A**, 225–230 (1977)

# Binary encounter approximation theory for stripping

R. P. SINGHAL AND VIR SINGH
*Physics Section, Institute of Technology,*
*Banaras Hindu University, Varanasi-221005*

(*Received* 14 *July* 1976)

Cross section formula for the stripping of electrons from slow moving atoms colliding with stationary target atoms have been derived using the binary encounter approximation theory (BEA). The Born approximation has been used assuming a screened Coulomb potential for electron atom scattering cross-section. Calculations are made for the stripping of hydrogen atoms incident on nitrogen.

## 1. INTRODUCTION

When a projectile atom collids with a target atom the electrons may be ejected via three processes : (i) direct ionization of the target atom (ii) stripping of the incident projectile (iii) simultaneous ionization of projectile and target as given below :

$$A+B = A+B^{+}+e \qquad \ldots \text{(i)}$$

$$A+B = A^{+}+B+e \qquad \ldots \text{(ii)}$$

$$A+B = A^{+}+B^{+}+2e \qquad \ldots \text{(iii)}$$

Where A is the projectile atom and B is the target atom. Flannery (1971) has calculated the differential ionization cross-section for the process (i) using the the binary encounter approximation (BEA) theory. But for the processes (ii) and (iii) no simple theoretical formulations for the differential ionization cross-sections are available. In the present work we have derived total and differential cross section formulae for the stripping of electrons via process (ii) from slow moving projectiles incident on stationary target atoms using the approach of Flannary.

## 2. THEORY

Consider the collision between a hydrogen like atom with it's centre of mass moving with velocity $\boldsymbol{V}$ and a target atom which is assumed at rest throughout the collision. Let the velocities of the projectile nucleus and the electron attached to it be $\boldsymbol{v}_2$ and $\boldsymbol{v}_1$ and let their masses be $m_1$ and $m_1$ respectively. In the binary encounter approximation it is assumed that the electron attached to the projectile is elastically scattered from the target causing the rotation of the electron's velocity $\boldsymbol{v}_1$ to $\boldsymbol{v}_1'$ through the spherical angle $(\theta, \phi)$ taken according to a right hand coordinate system in which $\boldsymbol{v}_1$ lies along the $z$-axis. The result of the collision is to change the internal energy of relative motion of masses $m_1$ and $m_2$.

Vrions (1969) has already shown that ionization of the projectile takes place if the change in the internal energy of the projectile is greater than the binding energy $U_i$. Thus one can write.

$$\tfrac{1}{2}m_{12}(\boldsymbol{v}_1'-\boldsymbol{v}_2)^2-\tfrac{1}{2}m_{12}(\boldsymbol{v}_1-\boldsymbol{v}_2)^2 = U_i+\tfrac{1}{2}m_{12}(\boldsymbol{v}'-\boldsymbol{v}_2)^2 \tag{1}$$

where $\boldsymbol{v}'$ is the velocity of the ejected electron, being in the same direction as $\boldsymbol{v}_1'$ and $m_{12}$ is the reduced mass of the projectile.

The cross section for the process in which $\boldsymbol{v}_1$ is rotated to within solid angle $d(\cos\psi)d\phi$ about $(\psi, \phi)$ and in which $\lambda$ defined by

$$v_1^2 = m^2u^2+V^2+2muV\lambda \tag{2}$$

is within interval $d\lambda$ around $\lambda$ is

$$dQ = \tfrac{1}{2}\sigma(v_1, \psi)d(\cos\psi)d\phi d\lambda \tag{3}$$

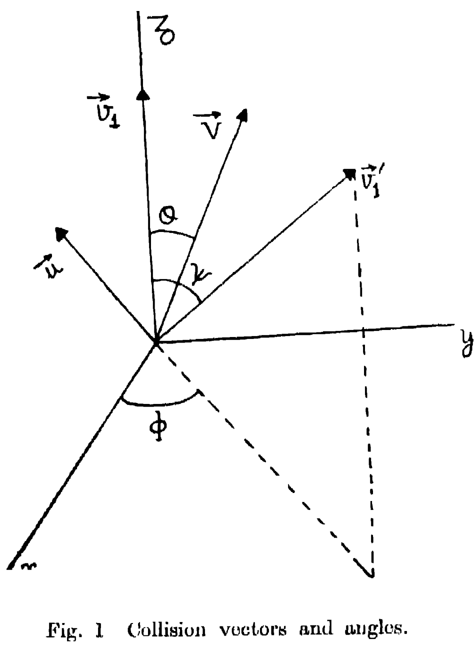

Fig. 1 Collision vectors and angles.

Here $\sigma(v_1, \psi)$ represents the differential cross section for elastic scattering between the electron and target atom, $m = m_2/(m_1+m_2)$ and $\boldsymbol{u} = \boldsymbol{v}_1-\boldsymbol{v}_2$ is the relative velocity vector of the electron with respect to nucleus. Substituting for $\boldsymbol{v}_2$ from

$$\boldsymbol{V} = (m_1\boldsymbol{v}_1+m_2\boldsymbol{v}_2)/(m_1+m_2)$$

and taking the vectors $\boldsymbol{u}$ and $\boldsymbol{V}$ in the $x-z$ plane as shown in figure 1, one can rewrite equation (1) as follows

$$X-T-m_1(v_1-v')(Y\cos\psi+Z\sin\psi\cos\phi) = 0 \qquad \ldots \quad (4)$$

where $T$ is the kinetic energy of the ejected electron and

$$X = \tfrac{1}{2}m_1(v_1^2-u^2)-U_i/m$$
$$Y = V\cos(\theta)/m-m_1v_1/m_2$$
$$Z = V\sin(\theta)/m$$

$\theta$ is the angle between $\boldsymbol{v}_1$ and $\boldsymbol{V}$.

Differentiating eq. (4) w.r.t. $\phi$ keeping $v_1$ and $\psi$ fixed one obtains

$$\frac{d\phi}{dT} = \frac{-2(X+T-m_1v_1v')}{m_1^2(v_1-v')^2v'(Y^2+Z^2)^{\frac{1}{2}}\{R^2(\psi)\}^{\frac{1}{2}}} \qquad \ldots \quad (5)$$

The factor of two arises since from eq. (4) one notes that a given $T$ is passed through twice when $\phi$ rotates from zero to $2\pi$. In eq. (5).

$$R^2(\psi) = -\cos^2\psi+b\cos\psi+C$$

with $b$ and $c$ defined as below

$$b = \frac{2(X-T)Y}{m_1(v_1-v')(Y^2+Z^2)}$$

$$c = \frac{m_1^2(v_1-v')^2Z^2-(X-T)^2}{m_1^2(v_1-v')^2(Y^2+Z^2)}$$

To calculate the elastic scattering cross section $\sigma(v_1, \psi)$ we have assumed screened Coulomb potential of the form $V(r) = \frac{Z_Ne^2}{r}\exp(-r/R)$. $Z_N$ is the atomic number of the target atom and $e$ is electronic charge. $R$ is given by

$$R = a_0Z_N^{-1/3}, \; a_0 \text{ being the Bohr radius.}$$

Born approximation then yields (Goldberger & Watson (1964))

$$\sigma(v_1, \psi) = \frac{Z_Ne^4}{m_1^2v_1^4(\theta_g-\cos\psi)^2} \qquad \ldots \quad (6)$$

where

$$\theta_g = 1+\tfrac{1}{2}\left(\frac{\hbar}{m_1v_1R}\right)^2$$

The zeroes of $R^2(\psi)$ that represent the limits to $\psi$ for fixed $v_1$ are given by

$$\cos\psi \pm = \frac{-b \pm (b^2+4c)^{\frac{1}{2}}}{} \quad \ldots \quad (7)$$

Eq. (3) can now be integrated over $\psi$ to yield

$$\frac{d\phi}{dT} = \frac{Z_N{}^2 e^4 \pi}{m m_1{}^4 v' V u} \int^{v_1+} \frac{|X+T-m_1 v_1 v'|(\theta_g - b/2)}{(v_1-v')^2 v_1{}^3 (Y^2+Z^2)^{\frac{1}{2}} (\theta_g{}^2 - b\theta_g - c)^{3/2}} dv_1 \quad \ldots \quad (8)$$

where for $d\lambda$ we have substituted from (2). $d\lambda = \dfrac{v_1 dv_1}{muV}$

The limits $v_1\pm$ are given by

$$mu - V \leqslant v_1 \leqslant mu + V$$

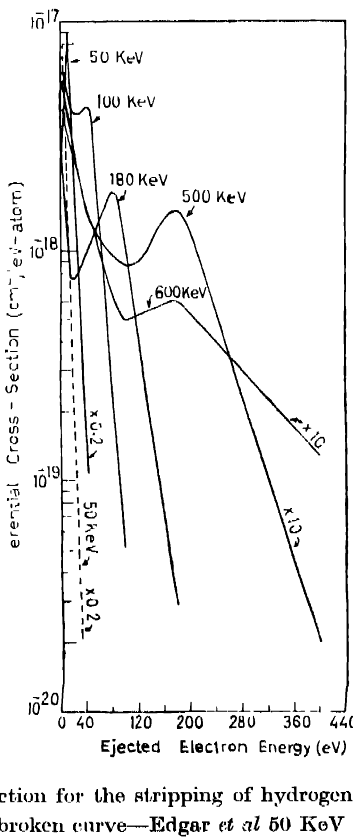


Fig. 2. Differential cross section for the stripping of hydrogen atoms incident on nitrogen. Full curve-present; broken curve—Edgar *et al* 50 KeV curves are 0.2 times and 500 and 600 KeV curves are 10 times.

together with the condition of reality of $\psi^{\pm}$ i.e. $b^2+4c \geq 0$. The roots of $v_1$ from $b^2+4c = 0$ are difficult to obtain in general. However under the valid assumption $m_1 << m_2$, roots are easily obtained. It then follows that

$$v_1^- = \max[((u^2+u_0^2+(v'+V)^2)^{\frac{1}{2}}-V), |u-V|]$$
$$v_1^+ = u+V$$

where $$u_0^2 = 2U_i/m_1.$$

Assuming an isotropic electron relative velocity distribution $f(u)du$, total cross section for the stripping of projectile is given-by

$$Q_{stripping} = \int_0^{\infty} f(u)du \int_{v'=0}^{v'_{max}} \frac{dQ}{dT} \frac{dT}{dv'} dv' \qquad \ldots \quad (9)$$

where, under the condition $m_1$ $(4V^2+4uV-u_0^2)^{\frac{1}{2}}-V$

## 3. Results and Discussion

We have performed calculations for the differential and total cross sections for the stripping of hydrogen atoms (10-600 keV) colliding with nitrogen atom.

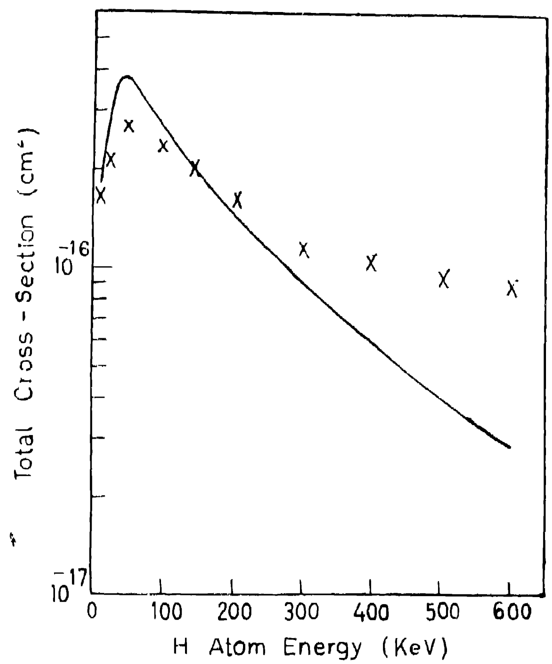


Fig. 3. Total cross section for the stripping of hydrogen atoms incident on nitrogen. Full curve—present; Cross (X) Barnott and Reynolds.

Hydrogenic velocity distribution function $f(u)$ has been used to calculate cross section in eq. (9). The results are shown in figures 2 and 3 The results of our total stripping cross sections have been compared with the experimental data of Barnett and Reynolds (1958). It may be observed that at low projectile energies our results are in good agreement with the experimental observations However at higher projectile energies there is deviation which can be understood since we have taken the target atom at rest throughout the collision, an assumption which may not be good at these energies due to the target recoil. This simplification has been introduced since for slow collisions where stripping process is dominant (Edgar *et al* (1973)) we can assume the target at rest. There is no experimental data available to compare the differential cross sections. However a knowledge of these cross sections is necessary to calculate energy loss and secondary electron spectrum due to the impact of protons in the atmosphere. It may be mentioned here that Edgar *et al* have arbitrarily assumed an exponential shape for the differential cross-section. The present study leads to more exact analysis of the differential cross-section for the stripping process. The application of the present work in atmospheric studies is in progress and the results will be published elsewhere.

## 4. Acknowledgment

One of us (V.S) is grateful to the C.S.I.R., India for providing a Junior Research Fellowship.

## References

Barnett C. F. & Reynolds H. K. 1958 *Phys Rev.* **109**, 355

Edgar B. C., Miles W. T. & Green A. E. S. 1973 *J. Geophys. Res.* **78**, 6595.

Flannery M. R. 1971 *J. Phys. B. Atom molec. Phys.* **4**, 892.

Goldberger M. L. & Watson K. N. 1964 *Collision Theory* (New York : John Wiley & Sons) 95.

Vriens L. 1969 Case studies in *Atomic Collision Physics I*, eds. E. W. McDaniel and M. R. C. McDowell (North-Holland : Amsterdam), Ch. 6.